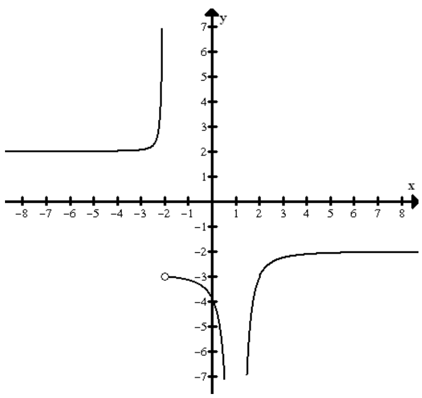
Math 1A Midterm 1 Review Answers

[1] $\dfrac{1}{2}$

[2] -6 meters per second

[3]



[4] Since $-1 \le \cos\dfrac{1}{x^2} \le 1$ for all x, therefore $-x^4 \le x^4\cos\dfrac{1}{x^2} \le x^4$ for all x.

And since $\lim\limits_{x\to 0}(-x^4) = \lim\limits_{x\to 0} x^4 = 0$, by the Squeeze Theorem, $\lim\limits_{x\to 0} x^4 \cos\dfrac{1}{x^2} = 0$ also.

[5] [a] -7 [b] -5 [c] DNE

[6] -3

[7] $$\lim_{x\to 2}\frac{x^2 g(x)}{1+f(x)} = \frac{\lim\limits_{x\to 2} x^2 g(x)}{\lim\limits_{x\to 2}(1+f(x))} = \frac{\lim\limits_{x\to 2} x\cdot\lim\limits_{x\to 2} x\cdot\lim\limits_{x\to 2} g(x)}{\lim\limits_{x\to 2}1+\lim\limits_{x\to 2} f(x)} = \frac{2\cdot 2\cdot 4}{1+(-3)} = -8$$

[8] discontinuities at $x=-3$ and $x=3$

$\lim\limits_{x\to -3^-} f(x) = -\infty$, $\lim\limits_{x\to -3^+} f(x) = \infty$, $\lim\limits_{x\to 3^-} f(x) = -\infty$, $\lim\limits_{x\to 3^+} f(x) = \infty$

[9] [a] no such a [b] 1 [c] $x=-1$ removable, $x=2$ jump

[10] Let $f(x) = \cos 2x - x^2$. Since $\cos 2x$ (a trigonometric function) and x^2 (a polynomial function) are both continuous for all x, so is their difference $f(x) = \cos 2x - x^2$. Since $f(\pi) = 1-\pi^2 < 0 < 1 = f(0)$, by the Intermediate Value Theorem, there is a value c in the interval $(0, \pi)$ such that $f(c) = \cos 2c - c^2 = 0$, ie. $\cos 2c = c^2$. So the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.

[11] $x = \dfrac{1}{2}$, $y = \pm\dfrac{3}{2}$

[12] $$f'(-2) = \lim_{x\to -2}\frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2}\frac{x^3-3x+2}{x+2} = \lim_{x\to -2}\frac{(x+2)(x^2-2x+1)}{x+2} = \lim_{x\to -2}(x^2-2x+1) = 9$$

$$f'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} = \lim_{x\to -2}\frac{(-2+h)^3-3(-2+h)+2}{h} = \lim_{x\to -2}\frac{-8+12h-6h^2+h^3+6-3h+2}{h}$$

$$= \lim_{x\to -2}\frac{9h-6h^2+h^3}{h} = \lim_{x\to -2}(9-6h+h^2) = 9$$

[13] [a] $f(x) = \cos \pi x$, $a = -1$ [b] $f(x) = x^2 - x$, $a = -2$

[14] 1.5 feet per minute

[15] $y + 4 = 2(x - 2)$

[16] $f'(-2) < f'(4) < 0 < f'(2) < f'(-4)$

[17] [a] If the refrigerator temperature is $4^\circ C$, the meat will defrost in 6 hours.

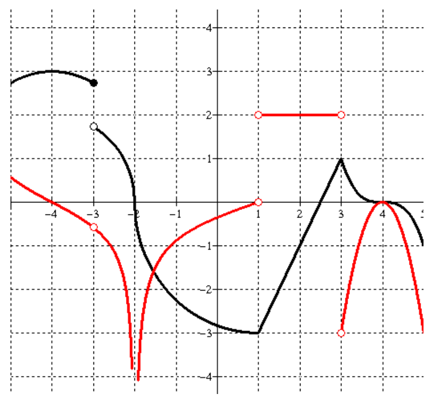
[b] If the refrigerator temperature is $4^\circ C$, the meat will defrost 1 hour sooner for each $1^\circ C$ increase in the refrigerator's temperature.

[c] No. The defrost time should always decrease if the refrigerator temperature increases. The meat will always defrost faster in a warmer refrigerator.

[18] [a] $f'(t) = \dfrac{1}{2(1-t)^{\frac{3}{2}}}$ [b] $g'(x) = \dfrac{8}{(2-x)^2}$

[19] [a] $x = -3$ (discontinuous)
$x = -2$ (vertical tangent line)
$x = 1, 3$ (cusps)

[b]



[20] Since the line $x - 2y = 6$ or $y = \frac{1}{2}x - 3$ is tangent to $y = f(x)$ at $x = 4$,

therefore the point of tangency is $\left(4, \frac{1}{2}(4) - 3\right)$ or $(4, -1)$.

That means $f(4) = -1$ and $f'(4) = \frac{1}{2}$.

Since $f'(4)$ exists, therefore f is differentiable at $x = 4$ (by the definition of "differentiable").

Since f is differentiable at $x = 4$, therefore f is continuous at $x = 4$ (by the "differentiability implies continuity" theorem).

Since f is continuous at $x = 4$, therefore $\lim_{x \to 4} f(x) = f(4) = -1$ (by the definition of "continuous at a point").